

Grupa 3

Zad 1

$$L := 5\text{m} \quad P_0 := 7\text{kN} \quad b := 12\text{cm} \quad h := 20\text{cm} \quad g := 3\text{cm}$$

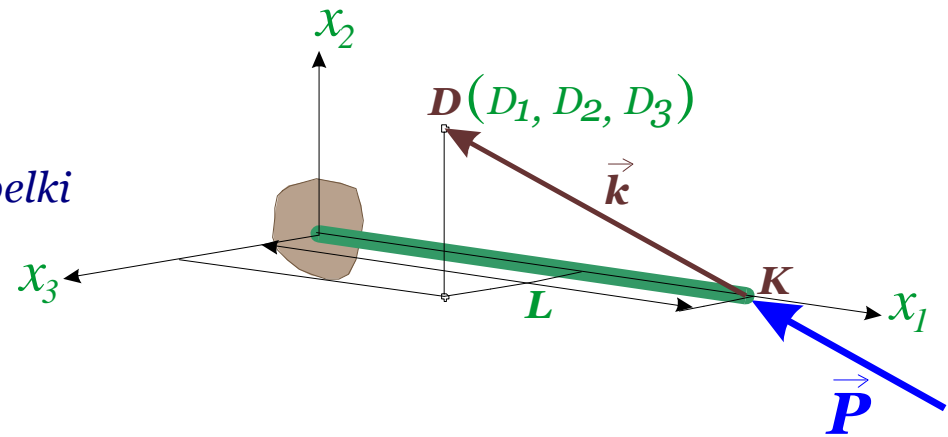
$$D := \begin{pmatrix} 2 \\ 7 \\ -3 \end{pmatrix} \text{m} \quad - \text{współrzędne punktu przez który przechodzi kierunek siły}$$

$$K := \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad - \text{współrzędne punktu } K, \text{ obciążonego końca belki}$$

$$k := D - K \quad - \text{wektor kierunkowy siły} \quad k = \begin{pmatrix} -3 \\ 7 \\ -3 \end{pmatrix} \text{m}$$

$$L_k := \sqrt{(k_1)^2 + (k_2)^2 + (k_3)^2} = 8.18535 \text{ m} \quad - \text{moduł (długość) wektora kierunkowego}$$

$$c := \frac{1}{L_k} \cdot k = \begin{pmatrix} -0.366508 \\ 0.855186 \\ -0.366508 \end{pmatrix} \quad - \text{kosinusy kierunkowe wektora siły } P$$



$$P := P_0 \cdot c \quad - \text{składowe wektora siły} \quad P = \begin{pmatrix} -2.566 \\ 5.986 \\ -2.566 \end{pmatrix} \cdot \text{kN}$$

$$\underline{N} := P_1 \quad T_2 := P_2 \quad T_3 := P_3$$

$$N = -2.56556 \cdot \text{kN} \quad T_2 = 5.9863 \cdot \text{kN} \quad T_3 = -2.56556 \cdot \text{kN}$$

$$M_2 := -T_3 \cdot L \quad M_3 := T_2 \cdot L$$

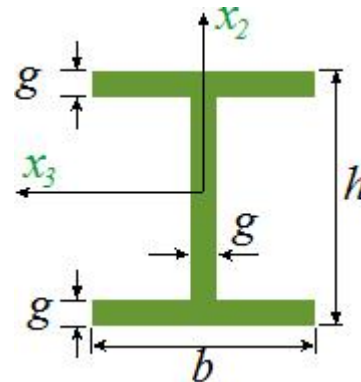
$$M_2 = 1.28278 \times 10^1 \cdot \text{kN} \cdot \text{m} \quad M_3 = 2.99315 \times 10^1 \cdot \text{kN} \cdot \text{m}$$

$$h_1 := h - 2g \quad b_1 := b - g$$

$$\underline{A} := h \cdot b - h_1 \cdot b_1 = 114 \cdot \text{cm}^2$$

$$J_3 := \frac{b \cdot h^3}{12} - \frac{b_1 \cdot h_1^3}{12} = 5942 \cdot \text{cm}^4$$

$$J_2 := \frac{b^3 \cdot g}{6} + \frac{h_1 \cdot g^3}{12} = 895.5 \cdot \text{cm}^4$$



Naprężenia w punkcie A

$$y := \frac{h}{2} \quad z := \frac{b}{2} \quad a2 := 2g \quad a3 := b$$

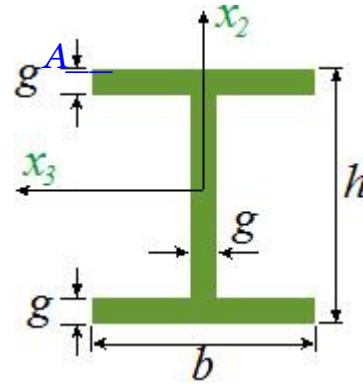
$$S3 := 0 \quad S2 := 0$$

$$\sigma_{11} := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = 35.351 \cdot \text{MPa}$$

$$\tau_{12} := \frac{T2 \cdot S3}{a3 \cdot J3} = 0.000 \cdot \text{MPa}$$

$$\tau_{13} := \frac{T3 \cdot S2}{a2 \cdot J2} = 0.000 \cdot \text{MPa}$$

$$\sigma_{\text{HMH}} := \sqrt{\sigma_{11}^2 + 3 \cdot (\tau_{12}^2 + \tau_{13}^2)} = 35.351 \cdot \text{MPa}$$

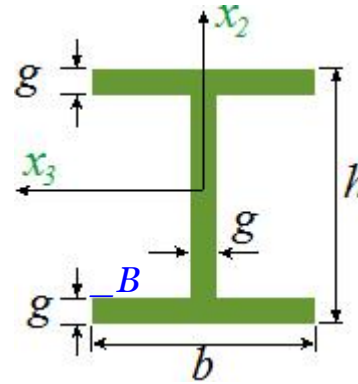


Naprężenia w punkcie B

$$y := \frac{-h_1}{2} \quad z := \frac{b}{2} \quad a_2 := 2g \quad a_3 := b$$

$$S_3 := b \cdot g \cdot \frac{(h-g)}{2} \quad S_3 = 306 \cdot \text{cm}^3$$

$$S_2 := 0 \quad S_2 = 0 \cdot \text{cm}^3$$



$$\sigma_{11} := \frac{N}{A} - \frac{M_3 \cdot y}{J_3} + \frac{M_2 \cdot z}{J_2} = 120.984 \cdot \text{MPa}$$

$$\tau_{12} := \frac{T_2 \cdot S_3}{a_3 \cdot J_3} = 0.257 \cdot \text{MPa}$$

$$\tau_{13} := \frac{T_3 \cdot S_2}{a_2 \cdot J_2} = 0.000 \cdot \text{MPa}$$

$$\sigma_{\text{HMH}} := \sqrt{\sigma_{11}^2 + 3 \cdot (\tau_{12}^2 + \tau_{13}^2)} = 120.985 \cdot \text{MPa}$$

Naprężenia w punkcie C

$$\underline{y} := \frac{h1}{2} \quad \underline{z} := \frac{-g}{2} \quad \underline{a2} := h \quad \underline{a3} := g$$

$$\underline{S3} := b \cdot g \cdot \frac{(h - g)}{2} \quad S3 = 306 \cdot \text{cm}^3$$

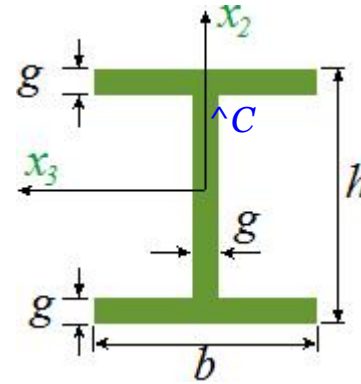
$$\underline{S2} := 2 \cdot g \cdot b1 \cdot \frac{(b1 + g)}{2} + h \cdot \frac{g^2}{8} \quad S2 = 346.5 \cdot \text{cm}^3$$

$$\underline{\sigma_{11}} := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = -56.973 \cdot \text{MPa}$$

$$\underline{\tau_{12}} := \frac{T2 \cdot S3}{a3 \cdot J3} = 1.028 \cdot \text{MPa}$$

$$\underline{\tau_{13}} := \frac{T3 \cdot S2}{a2 \cdot J2} = -0.496 \cdot \text{MPa}$$

$$\underline{\sigma_{\text{HMH}}} := \sqrt{\sigma_{11}^2 + 3 \cdot (\tau_{12}^2 + \tau_{13}^2)} = 57.007 \cdot \text{MPa}$$



Zad. 2

$$E := 15 \text{ GPa} \quad L := 6 \text{ m} \quad b := 18 \text{ cm} \quad h := 20 \text{ cm} \quad g := 3 \text{ cm}$$

$$\mu := \mu_0 \quad L_w := \mu \cdot L \quad b_1 := b - 2g \quad h_1 := h - 2g$$

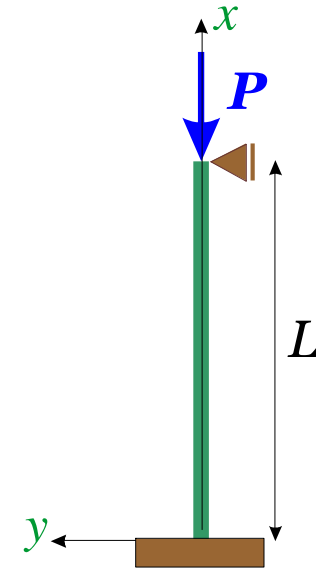
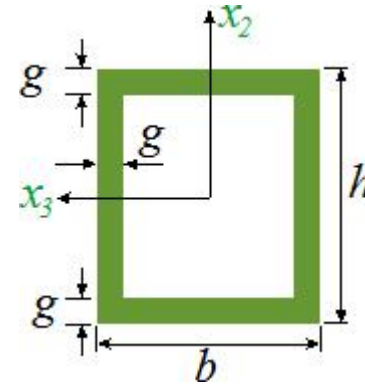
$$J_3 := \frac{b \cdot h^3}{12} - \frac{b_1 \cdot h_1^3}{12} = 9.256 \times 10^3 \cdot \text{cm}^4$$

$$J_2 := \frac{h \cdot b^3}{12} - \frac{h_1 \cdot b_1^3}{12} = 7.704 \times 10^3 \cdot \text{cm}^4$$

$$J := \min(J_2, J_3) = 7.704 \times 10^{-5} \text{ m}^4$$

$$P_{kr} := \frac{\pi^2 E \cdot J}{L_w^2} = 648.122 \cdot \text{kN}$$

$$\mu_0 := 0.699156$$



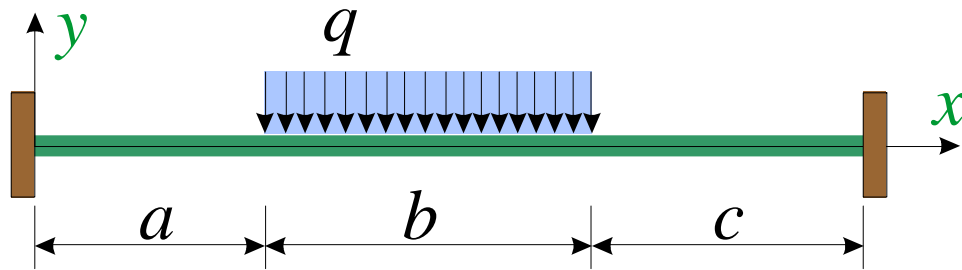
Zad. 3

$$q := 3 \frac{\text{kN}}{\text{m}} \quad a := 4\text{m} \quad \underline{b} := 3\text{m} \quad c := 3\text{m} \quad \underline{L} := a + b + c$$

$$\xi_1 := \frac{a}{L} = 0.4$$

$$\xi_2 := \frac{a+b}{L} = 0.7$$

$$H0001(\xi) := -\xi^2 \cdot (1 - \xi)$$

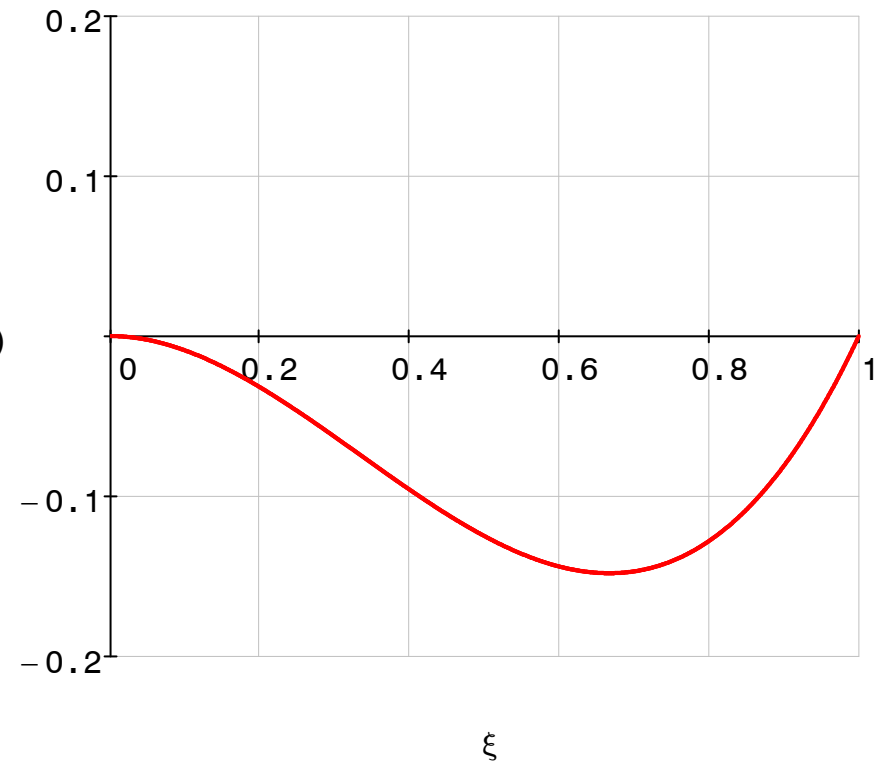


H0001(ξ)

Równanie pracy wirtualnej

$$-M \cdot 1 - q \cdot L^2 \cdot \int_{\xi_1}^{\xi_2} H0001(\xi) d\xi = 0$$

$$\underline{R} := -q \cdot L^2 \cdot \int_{\xi_1}^{\xi_2} H0001(\xi) d\xi = 11.8125 \text{m} \cdot \text{kN}$$



Definicja wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej

$$H1000(\xi) := 1 - 3 \cdot \xi^2 + 2 \cdot \xi^3 \quad H0010(\xi) := \xi^2 \cdot (3 - 2 \cdot \xi)$$

$$H0100(\xi) := \xi \cdot (1 - 2 \cdot \xi + \xi^2) \quad \underline{H0001}(\xi) := -\xi^2 \cdot (1 - \xi)$$

Definicja wielomianów Hermite'a dla belki zamocowanej przegubowo:

na prawej podporze - G(ξ)

$$G1000(\xi) := 1 - \frac{3}{2} \cdot \xi^2 + \frac{1}{2} \cdot \xi^3$$

$$G0100(\xi) := \frac{\xi}{2} \cdot (2 - 3 \cdot \xi + \xi^2)$$

$$G0010(\xi) := \frac{\xi^2}{2} \cdot (3 - \xi)$$

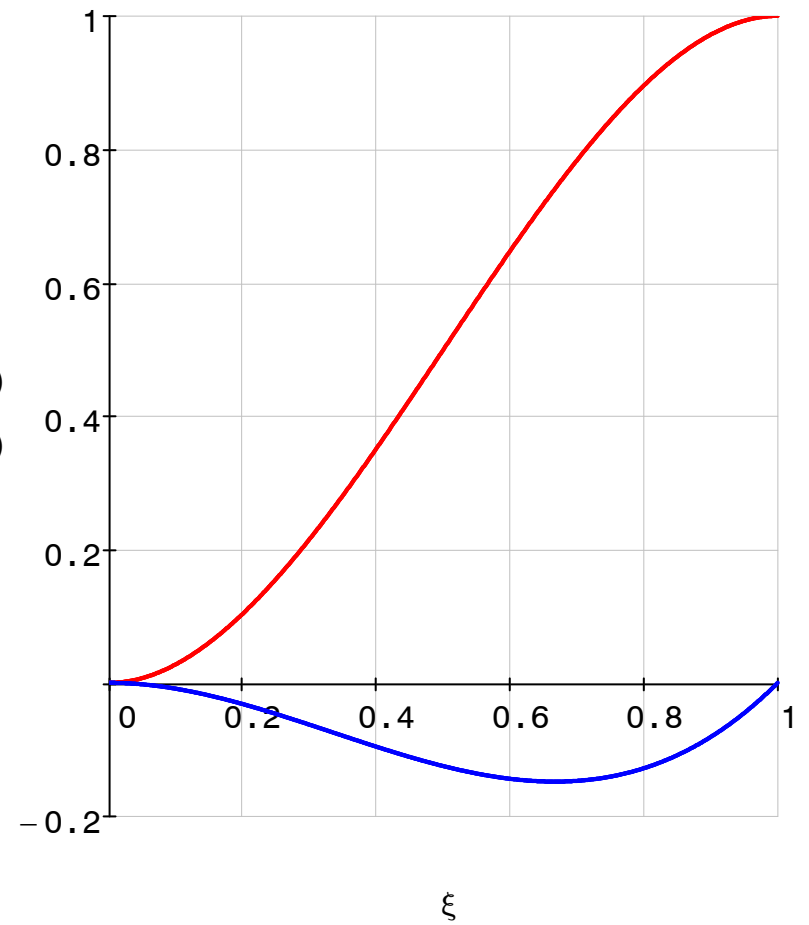
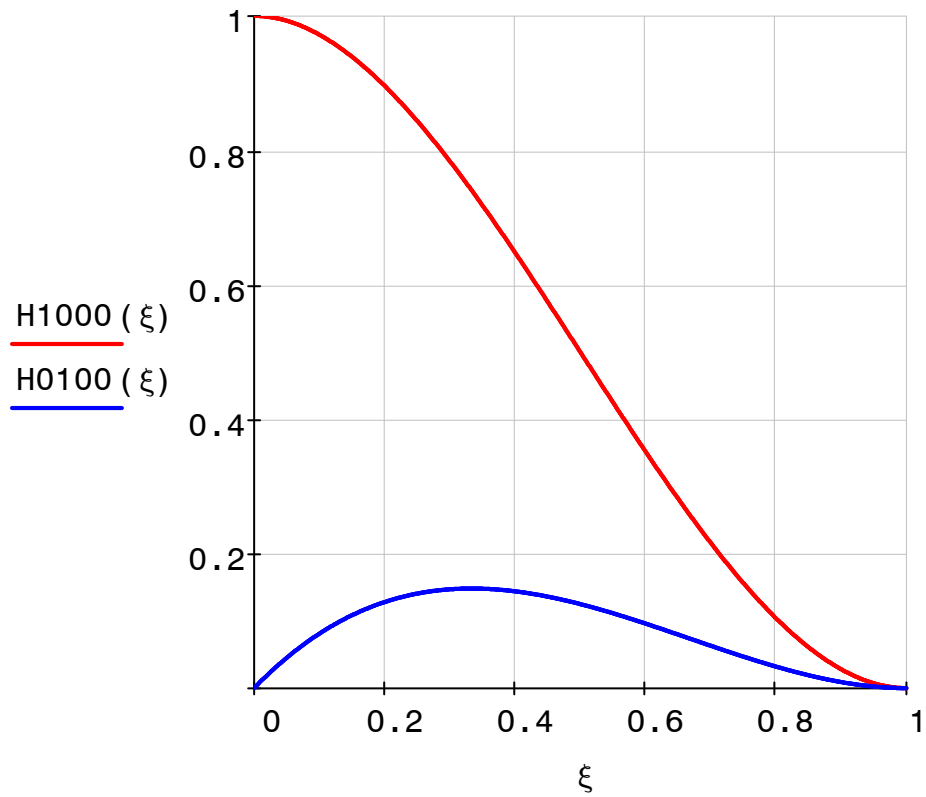
na lewej podporze - K(ξ)

$$K1000(\xi) := \frac{1}{2} \xi^3 - \frac{3}{2} \xi + 1$$

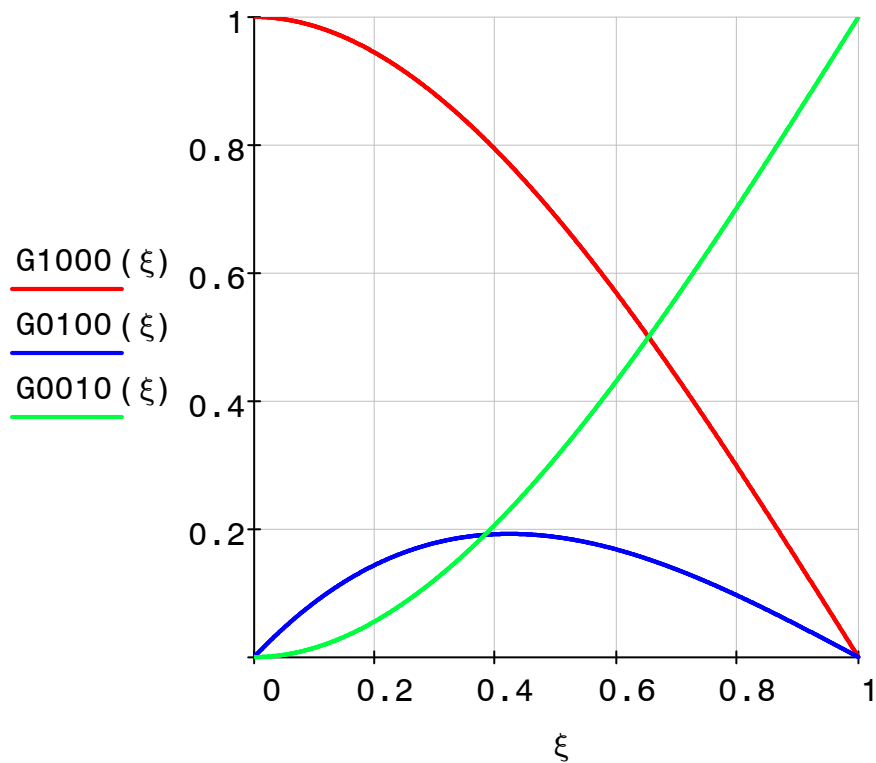
$$K0010(\xi) := \frac{\xi}{2} \cdot (3 - \xi^2)$$

$$K0001(\xi) := \frac{\xi}{2} \cdot (\xi^2 - 1)$$

Wykresy wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej



Wykresy wielomianów Hermite'a dla belki zamocowanej przegubowo:
na prawej podporze - $G(\xi)$



lewej podporze - $K(\xi)$

